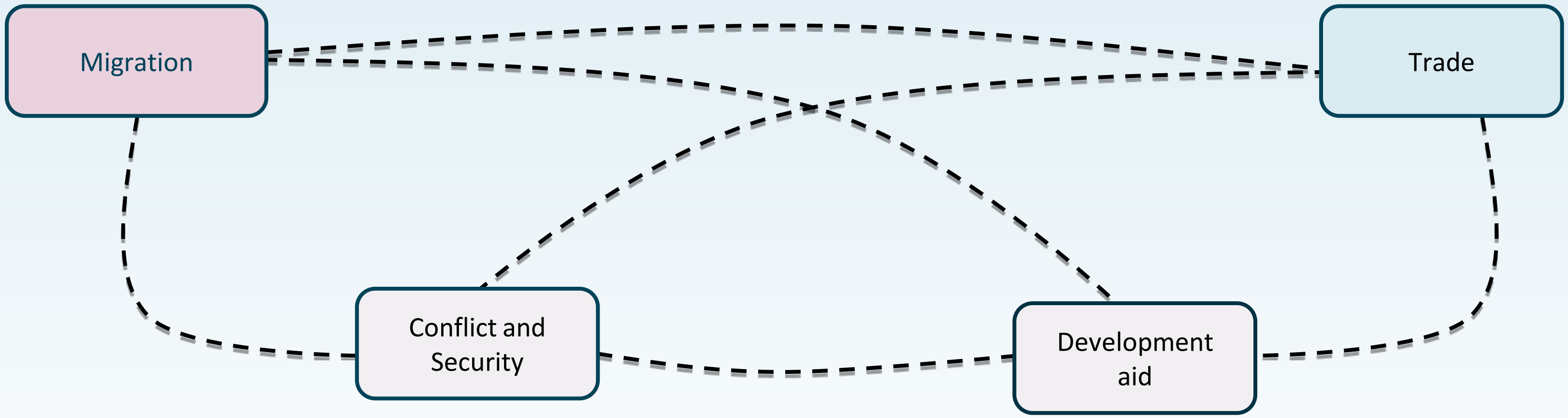
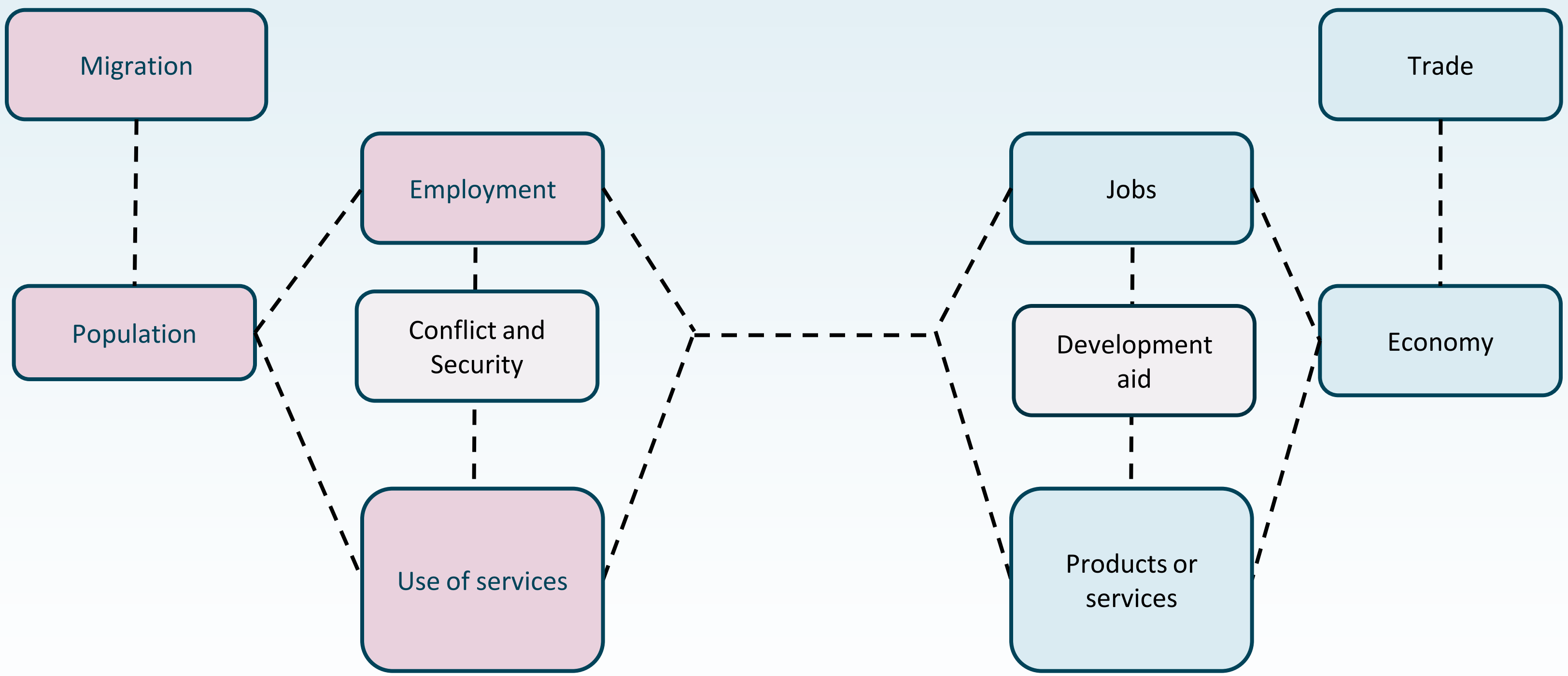
Using complex systems methods to model global dynamics

Hannah Fry

The global dynamics project



The global dynamics project



Discrete choice

Migration	People choose to migrate based on population, proximity, economy, job markets
Trade	Consumers buy goods because of price and ease of access
Conflict and Security	Nations place arms or troops in line with threat and strategic value
Development aid	Aid is offered based on diplomatic relationship, economic relationship and level of urgency

Discrete Choice

- The set of alternatives must be exhaustive
- The alternatives must be mutually exclusive
- The set must contain a finite number of alternatives

The McFadden conditional logit model

Representative tastes of the population

Variation in taste of the individual

Utility of individual i choosing alternative j

$$U_{ij} = V_{ij} + \epsilon_{ij}$$

Functions of the attributes of the individual and the alternatives

$$P(\text{individual } i \text{ chooses } j) = \frac{\exp\left(V_{ij}\right)}{\sum_k \exp\left(V_{ik}\right)}$$

Migration as a discrete choice problem

$$V_{ij} = \alpha \ln\left(P_j\right) - \beta d_{ij}$$

Distance between origin and destination

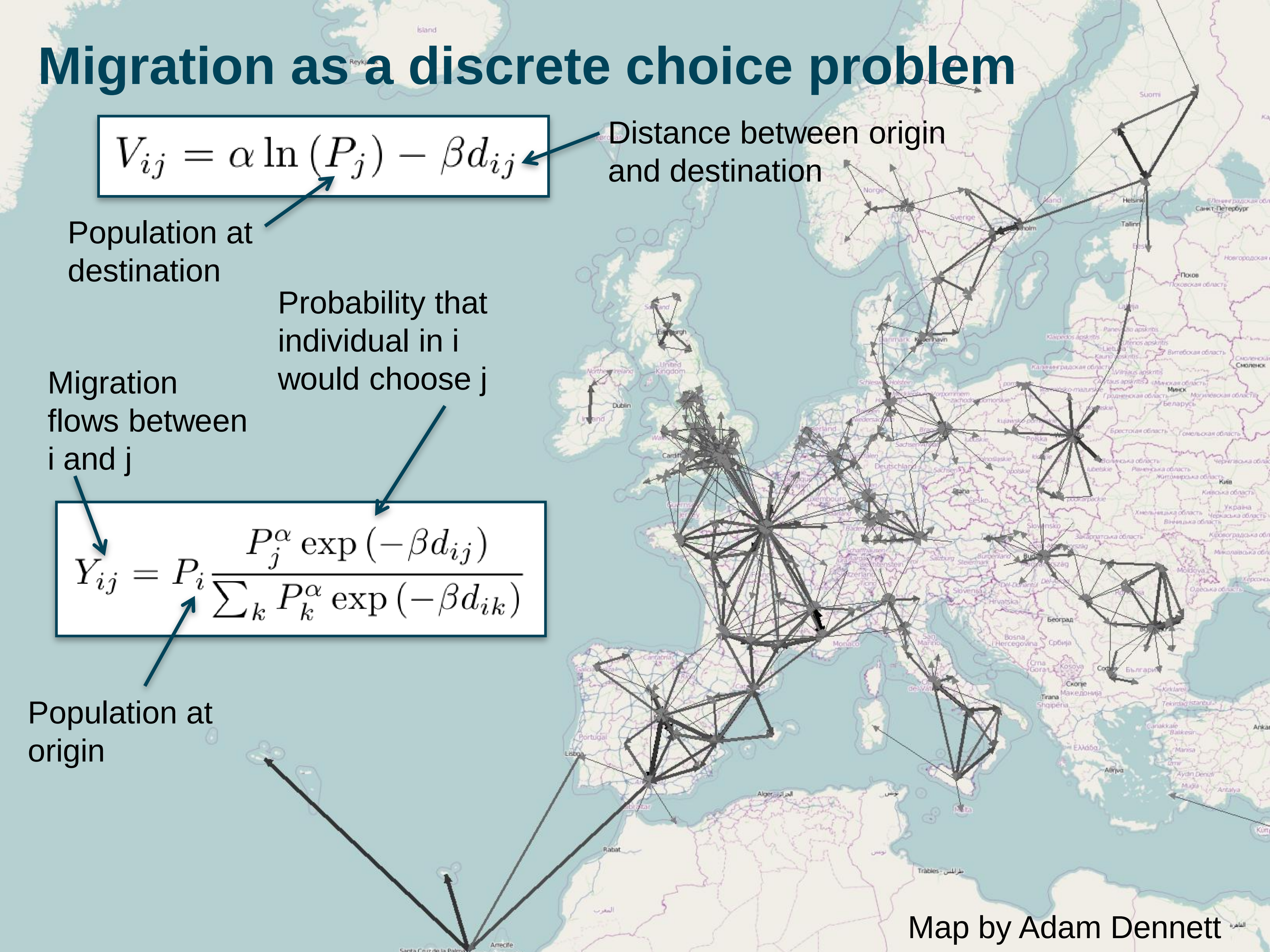
Population at destination

Probability that individual in i would choose j

Migration flows between i and j

$$Y_{ij} = P_i \frac{P_j^{\alpha} \exp\left(-\beta d_{ij}\right)}{\sum_k P_k^{\alpha} \exp\left(-\beta d_{ik}\right)}$$

Population at origin



Map by Adam Dennett

Other migration models

Conditional logit model:

Entropy maximising spatial interaction

All these models can be considered algebraically equivalent.

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Overall level of migration

Push term for each region

Pull term for each region

Interaction between zones

$$Y_{ij} = \frac{kP_iP_j}{f(d_{ij})}$$

Embedding discrete choice models into dynamics

Example 1: the retail model

Flow of money from residential area i to retail centre j

Spending power of residential area i

$$Y_{ij} = \frac{X_i Z_j^{\alpha} e^{-\beta d_{ij}}}{\sum_k Z_k^{\alpha} e^{-\beta d_{ik}}}$$

Retail floorspace of centre j

$$\text{Profit} = \sum_i \frac{X_i Z_j^{\alpha} e^{-\beta d_{ij}}}{\sum_k Z_k^{\alpha} e^{-\beta d_{ik}}} - kZ_j$$

Running costs per unit floorspace

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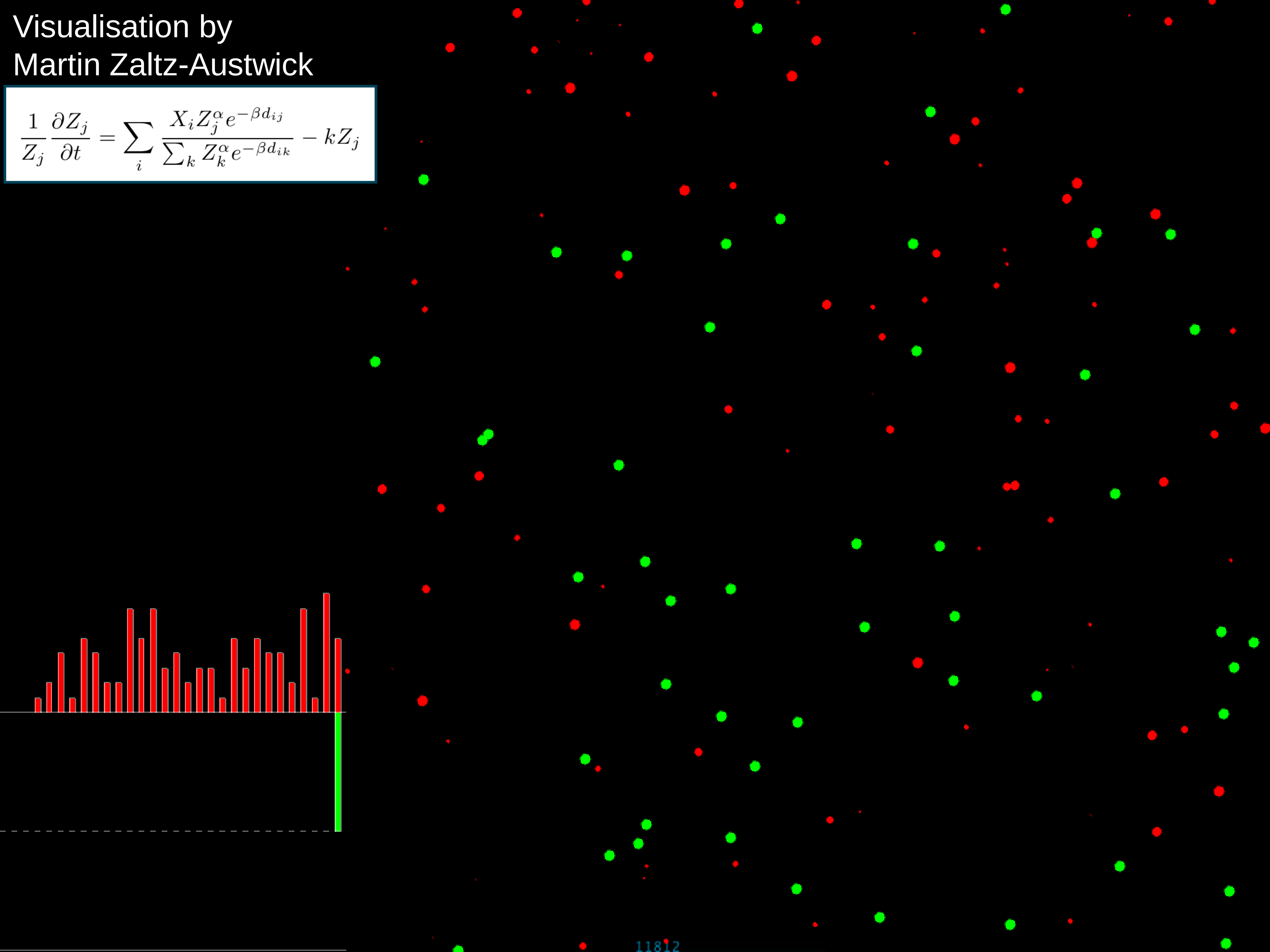
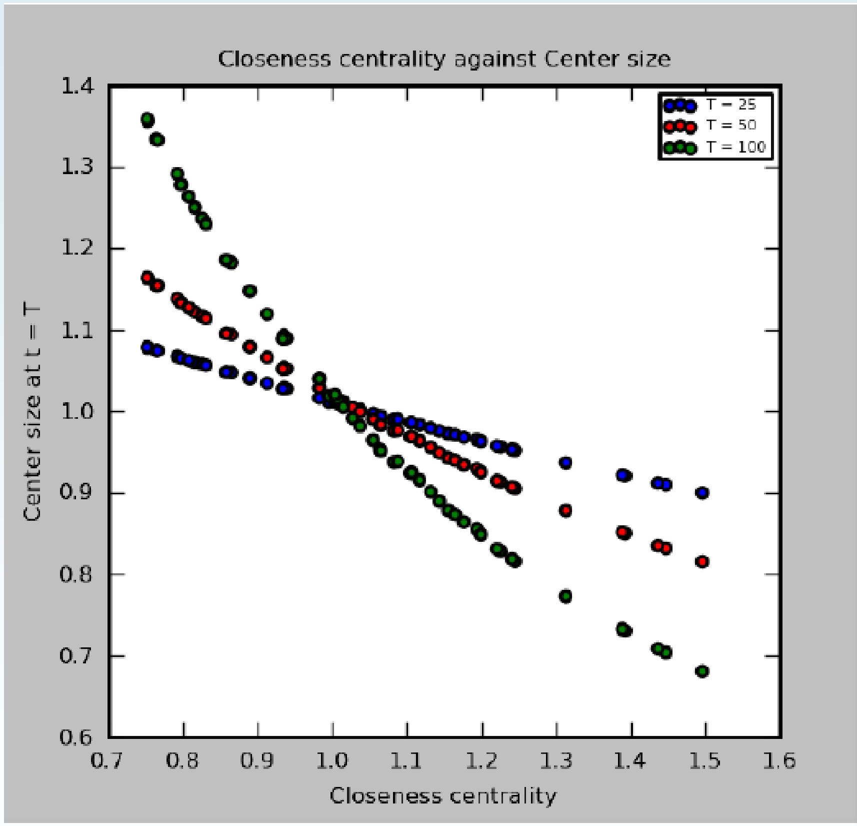
Visualisation by
Martin Zaltz-Austwick

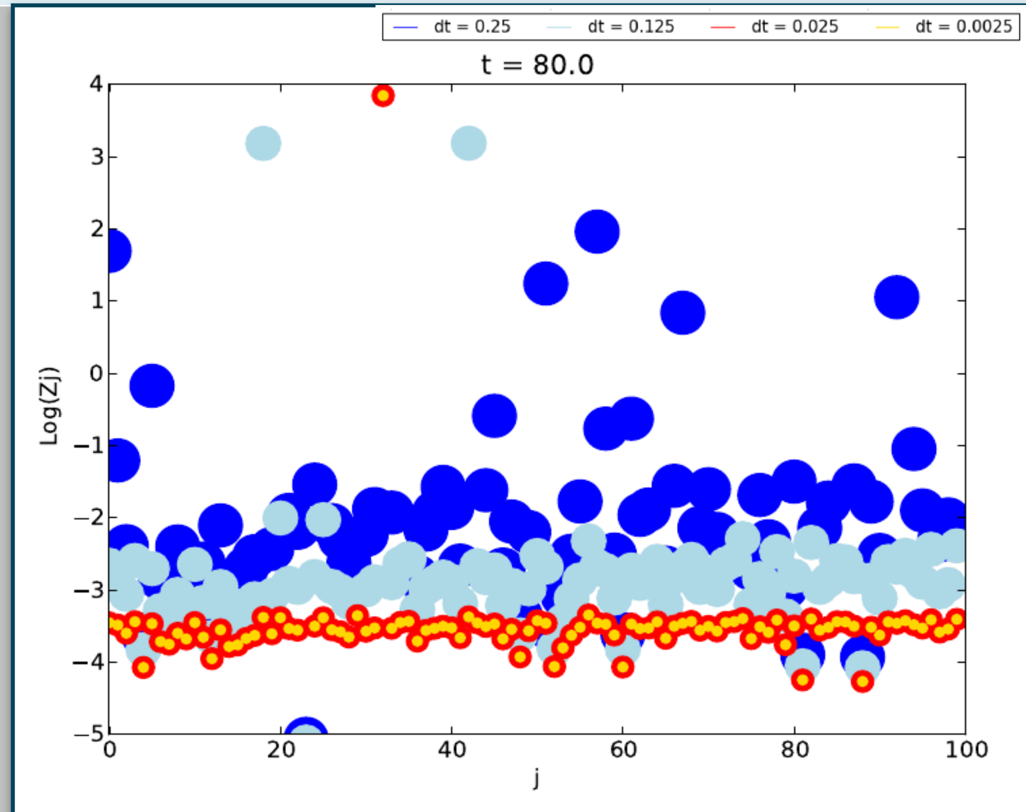
$$\frac{1}{Z_j}\frac{\partial Z_j}{\partial t} = \sum_i \frac{X_i Z_j^{\alpha} e^{-\beta d_{ij}}}{\sum_k Z_k^{\alpha} e^{-\beta d_{ik}}} - kZ_j$$



Embedding discrete choice models into dynamics

Example 1: the retail model

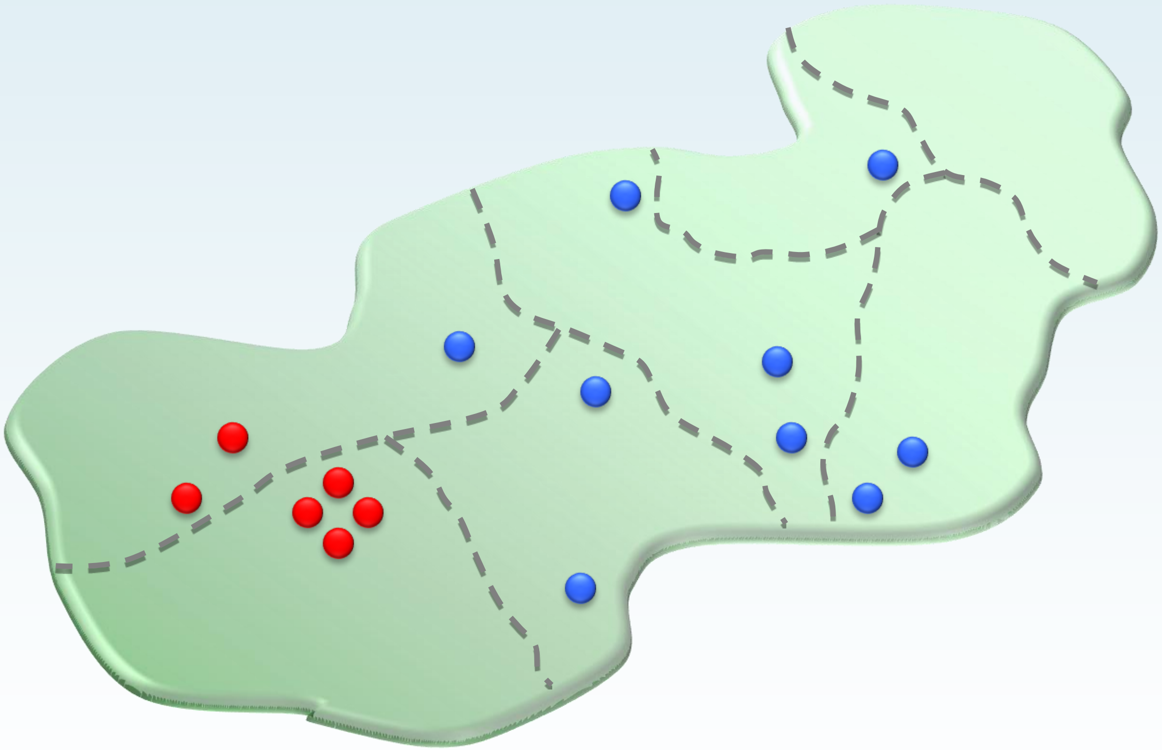




$$\frac{1}{Z}\frac{\partial Z}{\partial t} = \iint \frac{Z^{\alpha}(x,y,t)X(\bar{x},\bar{y})\exp\left(-\beta|r(x,y)-\bar{r}(\bar{x},\bar{y})|\right)}{\iint Z^{\alpha}(u,v,t)\exp\left(-\beta|r(u,v)-\bar{r}(\bar{x},\bar{y})|\right)\,\mathrm{d}u\,\mathrm{d}v}\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y} - kZ(x,y,t)$$

Embedding discrete choice models into dynamics

Example 2: A Blotto game of threat



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Embedding discrete choice models into dynamics

Example 2: A blotto game of threat

Red's troops in i

Blue's troops in j

Threat which Blue feels in j

$$T_j^B = \sum_i R_i \frac{B_j \exp\left(-\beta c_{ij}\right)}{\sum_k B_k \exp\left(-\beta c_{ik}\right)}$$

Accessibility of zone j from zone I (eg distance)

Normalise based on Blue's size in j:

$$\tau_j^B = \sum_i \frac{R_i \exp\left(-\beta c_{ij}\right)}{\sum_k B_k \exp\left(-\beta c_{ik}\right)}$$

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Embedding discrete choice models into dynamics

Example 2: A Blotto game of threat

$$\Pi^B = \sum_j \operatorname{sgn}\left(\tau_j^R - \tau_j^B\right) \left|B_j - R_j\right|^P$$

Zone 1	Zone 2
Blue: 1	Blue: 1
Red: 2	Red: 0

$$c_{ij} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$